BAIUST Academic Journal (BAJ), Volume 4, Issue 1, 2023: 14–25
https://doi.org/10.63307/BAJ.4.1.S2

A Comparative Analysis of Closed-loop Second-order MFB and Sallen & Key Lowpass Filters

Md. Jahid Hasan*, Nurul Bashar, Susmita Sarker, Sumiya Alam Lopa, and Tofayel Ahmed

Department of Electrical and Electronic Engineering, Bangladesh Army International University of Science and Technology (BAIUST), Cumilla, Bangladesh.
Email: jahid.eee@baiust.edu.bd

*Corresponding Author

Abstract: In contemporary times, the ubiquity of filters is undeniable. Active filters are frequently discussed and utilized among the various types of filters. This literature seeks to compare two types of second-order low pass filters, the second-order Butterworth Sallen & Key (VCVS) low pass filter and the Multiple Feedback (MFB) low pass filter, to shed light on the characteristics and performance of each filter. Upon analysis of the results, it was determined that the VCVS filter operates in a non-inverting mode, while the MFB filter operates in an inverting mode. Notably, when subject to an open loop noise-added signal, the VCVS filter output was observed to be 50-80% closer to the desired signal. Conversely, the MFB filter produced a severely distorted output under the same conditions. In a closed-loop PID-controlled method, both filters exhibited an output that was 90-95% closer to the desired output. However, the MFB low pass filter output in the closed-loop setting was considerably superior to that of the closed-loop VCVS filter. In light of these findings, it is evident that the selection of an appropriate filter must be made with consideration of a host of factors such as the cost of the filter, the desired signal quality, and the application's specific requirements. Ultimately, the appropriate filter selection could play a vital role in the successful execution of a given application or system.

Keywords**:** Low pass filter, sallen & key, butterworth, multiple feedback filter, closed-loop

1. Introduction

Signal processing is one of the most researchable fields in electronics engineering. The main task of this field is to analyze and manipulate the signal (Badajos et al., 2019). One of the main tasks of signal processing is filtration and filter designing. Noise has always been an undesirable part of any signal (Sudhakar et al., 2021). Noise is the one that contaminates the signal. So, for removing the noise, filter designing is important. Filter is a device that eliminates some of the undesired parts from a signal (Harry et al., 2022). It is said that a filter is used to separate the combined signal and restore the main signal (Smith, 1997). The filter can be of mainly two types: 1) the analog filter and 2) the digital filter. The analog filter can be of two types: 1) active filter and 2) passive filter. Whereas, the digital filter can be of two types: 1) IIR and 2) FIR. Whatever the filter is, each of the filters is of two types: 1) Low pass 2) High pass 3) Band pass 4) band stop. The analog filter is designed with hardware while the digital filter involves with microprocessor. In the analog signal category, the active filter needs an external source for filtration and it can amplify the filtered output. On the other hand, the passive filter does not need any external source for filtration and it cannot amplify the filtered output. In the digital filter category, the IIR gives output for the infinite duration of a dynamic system and FIR provides output for the finite duration of a dynamic system.

A low-pass filter passes the frequencies that are below the cutoff frequency and rejects the higher frequencies. On the other hand, a high pass filter rejects the frequencies that are below the cutoff frequency and passes the higher frequencies. A filter that allows (only a band of frequency range) the signals within the lower and upper frequency range is a band-pass filter and the opposite of it is the band-stop filter (*2nd Order Active Filters*, n. d.).

There are three types of filter response. 1) Butterworth response 2) Chebyshev response and 3) Bessel response. The Butterworth response is featured with flat amplitude and it is used when all frequencies of the pass band have the same gain. The Chebyshev response is featured with overshoot or ripple in passband and it is used in fewer pole, less complex circuitry. The Bessel response is featured with linear characteristics which means the phase shift increases linearly with frequency (*Nanopower Multiple-Input DTMOS*, n.d.).

In the realm of filters, the pole represents a pivotal factor that indicates the rate of roll-off and the sensitivity of the filter response. This fundamental component comprises a resistor and a capacitor that work in tandem to shape the filter's characteristics. The number of poles in a filter plays a crucial role in determining its overall performance. A higher number of poles translate to faster roll-off rates and higher sensitivity filters, as the effects of the resistors and capacitors are compounded. Additionally, poles are also commonly referred to as orders, with the number of poles corresponding to the order of the filter. A first-order filter, for instance, consists of a single capacitor and resistor combination. Conversely, a second-order filter comprises two capacitors and two resistors, thereby amplifying the filter's capabilities. This intuitive relationship between the number of poles and the filter order underpins the complex interplay between the filter's components, allowing engineers to design and customize filters for a broad range of applications. In summary, the pole's vital role in shaping the filter response, coupled with its relationship to the filter's order, highlights its importance in designing and developing effective filtering solutions. By carefully considering the number of poles in a filter, engineers can tailor their designs to meet specific performance requirements, ultimately leading to more efficient and effective systems (*Efficient Inverse Filters*, n.d.).

This literary work delves into a comprehensive comparison between two distinct second-order Butterworth low pass filters, namely the Sallen & Key and multiple feedback filters. Specifically, the focus is on evaluating the performance of these filters under two separate conditions: direct filtration and noise signal filtration. The closed-loop filtration method for both filters can improve the noise-added signal's output. Though these outputs are not exactly similar to the desired output, they are almost closer to the required output. Through this comparative analysis, readers can gain a deeper understanding of the inherent

https://doi.org/10.63307/BAJ.4.1.S2

strengths and limitations of each filter type and the effects of environmental factors on their efficacy. As such, this literature represents a valuable contribution to the ongoing efforts to refine and optimize filtering technologies across a wide range of domains. Overall, by examining the intricate workings of these two filters and their respective responses to different input signals, this study provides valuable insights into the underlying principles that drive their performance. These insights can help guide future research and development efforts aimed at improving the performance and efficiency of filtering systems in a variety of practical applications.

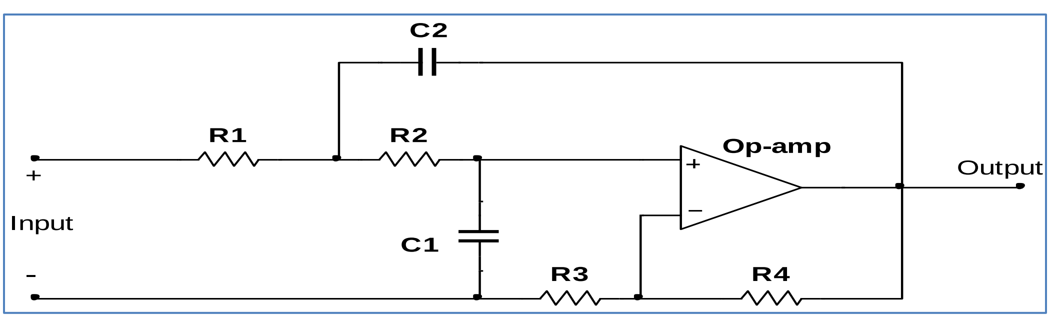
2. Methodology

The present literature is concerned with the comparison of the output generated by two types of second-order low-pass filters. To achieve this, the methodology is divided into three parts. Part I will go within the operation of the second order Butterworth Sallen & Key low pass filter in detail. Part II will go over the operation of the second order Butterworth multiple feedback low pass filter. Lastly, in Part III, the simulation technique for the stated filters will be thoroughly explained, highlighting the various steps involved in this process. The division of methodology into these parts will enable a comprehensive and systematic comparison of the performance of the two filters, thereby enabling the identification of the most suitable filter for a given application (Category, 2022).

2.1. VCVB Filter

Because of being an active filter, it must include an active component, such as an operation amplifier. Figure 1 depicts the Butterworth Sallen & Key low pass filter's electrical layout.

Figure 1
Circuit Diagram of Sallen and Key (VCVS) Low Pass Filter



Because capacitors are dependent on an alternating current system and its frequency, in low-frequency signals when the frequency is close to zero, the C1 and C2 capacitors form an open circuit and the R1 and R2 become series linked. So, the input signals are routed to the operational amplifier input port, the operational amplifier produces an amplified output, and when the capacitor short circuits, the signal is routed to ground, and the signal does not show in the output. R3 gets open-circuited and R4 turns short-circuited for unity gain. The transfer function of a second-order Butterworth Sallen & Key low pass filter can be obtained by Equation (1.1).

$$\frac{V_0}{V_i} = \frac{1}{s^2(R_1C_2R_2C_1) + s(R_1C_2 + R_2C_1 + R_1C_1) + 1} \tag{1.1}$$

The cut-off frequency for Butterworth, Sallen & Key low pass filter can be found by equations (1.2) and (1.3) based on the pre-known value of resistors R_1, R_2, R_3, and R_4 and capacitors C_1 and C_2.

$$f_c = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} \tag{1.2}$$

$$\boldsymbol{K = 1 + \frac{R_4}{R_3}} \tag{1.3}$$

The values to be used of resistors R1, R2, R3 & R4 and capacitors C1 & C2 can be adjusted by using the formulas below to get the required amount of filtration.

$$\omega_C = 2.\pi.f_C \tag{1.4}$$

$$C_2 \cong \frac{10}{f_c}(\mu F) \tag{1.5}$$

$$C_1 < \frac{[a^2 + 4.b.(K-1)].C_2}{4.b} \tag{1.6}$$

$$R_1 = \frac{2}{(a.C_2 + \{[a^2 + 4.b(K-1)].C_2{}^2 - 4.b.C_2.C_1\}^{1/2})\omega_C} \tag{1.7}$$

$$R_3 = \frac{K.(R_1 + R_2)}{(K-1)} \tag{1.8}$$

$$R_4 = K.(R_1 + R_2) \tag{1.9}$$

$$R3 = \infty \;\&\; R4 = 0 \tag{1.10}$$

Where,

$a = 1.41421$,

$b = 1.00$

Equations (1.8) and (1.9) are used, when $K \neq 1$ and Equation (1.10) is used when $K = 1$.

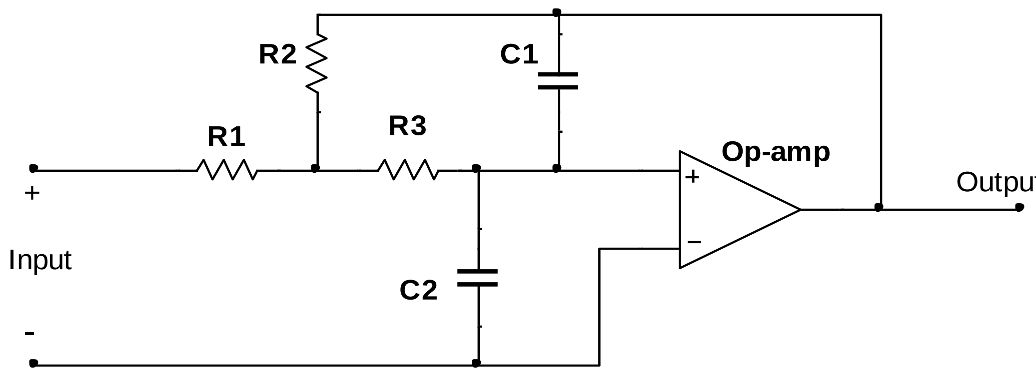
2.2. MFB FILTER

MFB filters also serve as active filters that employ an operational amplifier as an active component. At lower-frequency electrical signals when the frequency is almost 0, the capacitors C1 and C_2 remain open and the R_1 and R_3 become series connected inside the MFB low pass filter. As a consequence, the input signals are routed to the operational

https://doi.org/10.63307/BAJ.4.1.S2

amplifier input port, where the operational amplifier produces an increased output; at high frequencies, the capacitor forms a short circuit, and the signal is routed to ground, where it does not appear in the output.

Figure 2
Circuit Diagram of MFB Low Pass Filter



The transfer function of the MFB low pass filter is expressed in equation (2.1), and the shortened version of equation (2.1) is represented in equation (2.2).

$$\frac{V_0}{V_i} = \frac{-\dfrac{1}{C_1C_2R_1R_2}}{s^2 + s\dfrac{1}{C_1}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}\right) + \dfrac{1}{C_1C_2R_1R_3}} \tag{2.1}$$

$$H(s) = \frac{V_o}{V_i} = \frac{1}{As^2 + Bs + C} = \frac{K\omega_c^{\ 2}}{s^2 + \dfrac{\omega_c}{Q} + \omega_c^{\ 2}} \tag{2.2}$$

Where,

$A = (R_1R_3C_2C_1)$

$B = R_3C_1 + R_1C_1 + R_1R_3C_1 / R_2$

$C = \dfrac{R_1}{R_2}$

$$Q = \frac{\sqrt{R_3R_2C_2C_1}}{(R_2 + R_3 + |K|R_3)C_1} \tag{2.3}$$

$$K = -\frac{R_2}{R_1} \tag{2.4}$$

$$\omega_c = 2\pi f_c = \frac{1}{\sqrt{R_3 R_2 C_2 C_1}} \tag{2.5}$$

$$f_c = \frac{1}{2\pi\sqrt{R_3 R_2 C_2 C_1}} \tag{2.6}$$

For a desired level of filtration, the values of resistors R_1, R_2, R_3 & R_4, and capacitors C_1 & C_2 can be chosen from the equations below:

$$C_1 \cong \frac{10}{f_C}(\mu F) \tag{2.7}$$

$$C_2 < \frac{a.C_1}{4.b.(K+1)} \tag{2.8}$$

$$R_2 = \frac{2.(K+1)}{(a.C_1 + \{[a^2.C_1^{\ 2} - 4.b.C_1.C_2.(K+1)]^{1/2}\}\omega_C} \tag{2.9}$$

$$\boldsymbol{R_1 = \frac{R_2}{K}} \tag{2.10}$$

$$\boldsymbol{R_3 = \frac{1}{b.C_1.C_2.\omega_C^{\ 2}.R_2}} \tag{2.11}$$

Where,

$a = 1.41421$

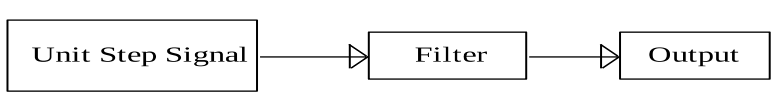
$b = 1.00$

2.3. SIMULATION PROCESS

Both filters were designed with a cutoff frequency of 50Hz in mind. Both the Sallen & Key and MFB low pass filter models were evaluated using the approaches specified underneath. As illustrated by Figure 3, a unit step input is initially sent directly into the filters.

Figure 3

Block Illustration of Unit Step Input to the Filter

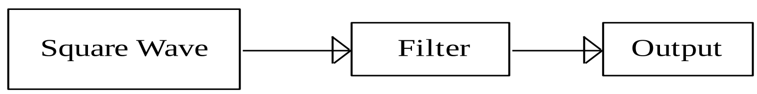


After that, for some realistic investigation, a 50Hz square wave is fed directly into the filter, as illustrated in Figure 4.

https://doi.org/10.63307/BAJ.4.1.S2

Figure 4

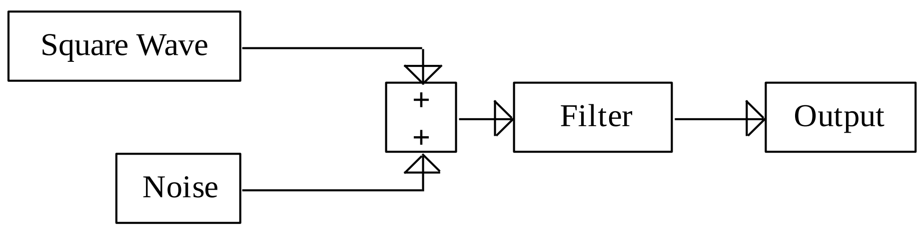
Block Illustration of Square Wave Input to the Filter



Then, for everyday life testing, an irregular noise signal was blended with the square signal and supplied to the filter, as shown in Figure 5. In this scenario, the filter was unable to adequately filter the signal, resulting in a somewhat distorted output.

Figure 5

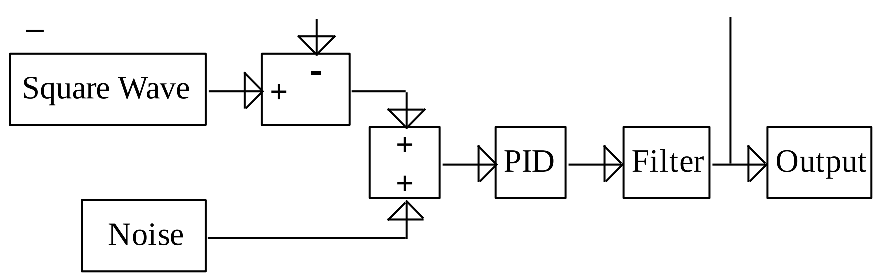
Block Illustration of 50Hz Square Wave with Noise Signal Input to the Filter



An adjusted PID control unit is then utilized before the filter circuits under the negative feedback method that controls the output, as shown in Figure 6. The output was likewise distorted, but it was almost identical to the original output, and it was DC superimposed.

Figure 6

Block Diagram of 50Hz Square Wave with Noise Signal Input to the Filter with PID Controller



3. Result and Analysis

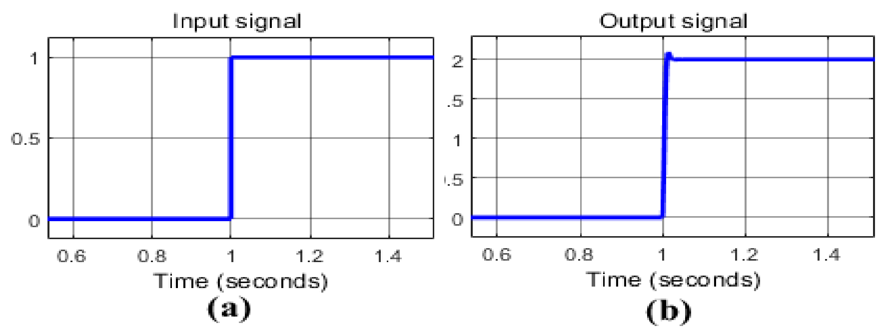
The result section will also be discussed in two parts. Part 1 will cover the Sallen & Key filter result analysis and Part 2 will cover the MFB filter result analysis. The simulation was done in a MATLAB environment.

3.1. VCVB Filter

This filter was developed with a cutoff frequency of 50Hz in mind. A unit step signal was initially fed into the second-order Sallen & Key filter. Figure 7 depicts the input and output response graph for the filter's unit step signal.

Figure 7

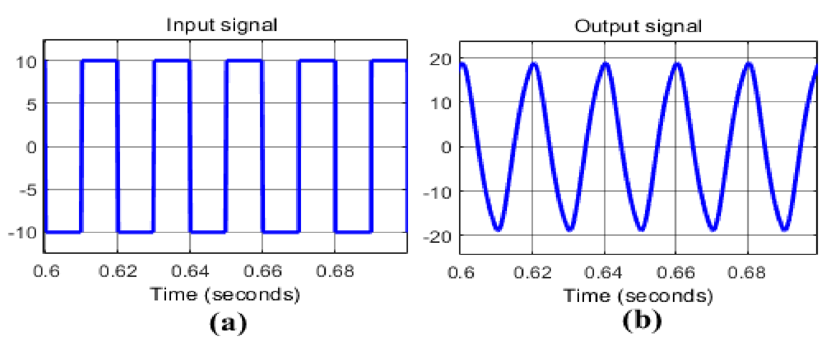
Input and Output Response of Sallen & Key Second-Order Filter for Unit Step Signal Input. (a) Input Signal, (b) Output Signal



The resultant signal of the second-order Sallen & Key filter displays a transient characteristic with overshoot for unit step input. The 50Hz square wave was then fed into the second-order Sallen & Key filter for testing. Figure 8 depicts the input and output response graph for the filter's 50Hz square wave signal.

Figure 8

Input and Output Response of Sallen & Key Second-Order Filter for Square Wave Input. (a) Input Signal, (b) Output Signal



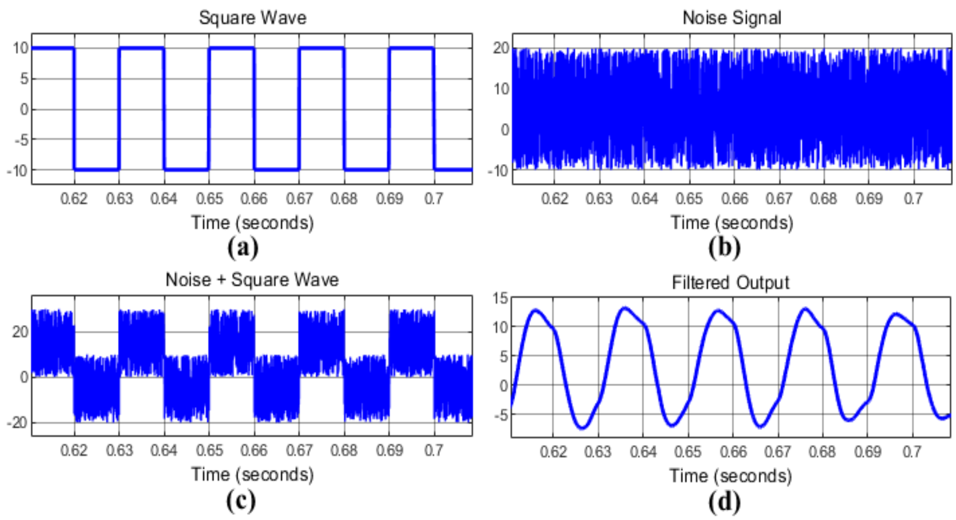
Because a square wave is a composite of many sinusoidal harmonics of the primary frequency, a 50Hz square wave will contain a primary frequency of 50Hz and harmonics of 100Hz, 150Hz, 200Hz, and many more integer multiples of 50Hz sine waves. The second-order Sallen & Key filter effortlessly isolated the 50Hz signal while blocking out the other frequencies.

https://doi.org/10.63307/BAJ.4.1.S2

A square wave signal at 50Hz was combined with a random noise signal for practical testing purposes. The composite signal that resulted was then sent to the filter circuit. Figure 9 illustrates the filter circuit's reaction to this input as well as its output.

Figure 9

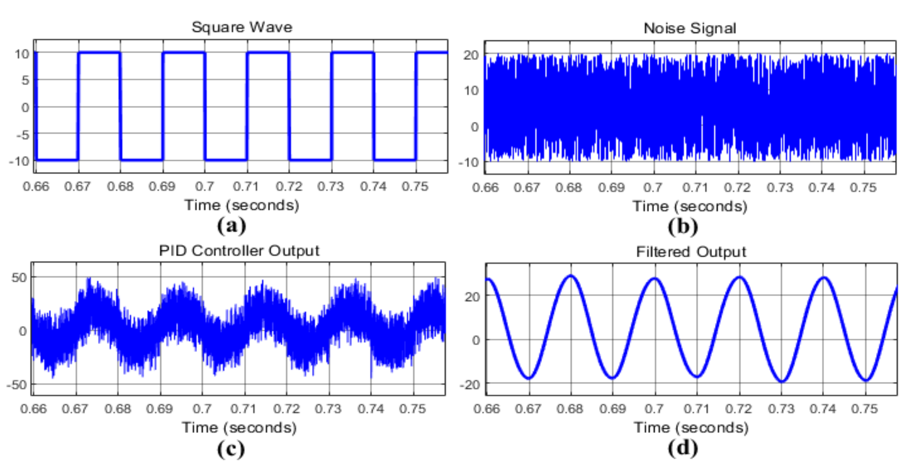
(a) Input Signal, (b) Random Noise Signal, (c) Noise Signal Added with the Square Wave, (d) Filter Output Signal for 50Hz Square Wave Input To VCVB Filter



When random noise is introduced, the Sallen & Key filter attempts to reject extraneous signals efficiently, leading to distortions in the filtered output signals. As a result, a PID controller was added to the system. Figure 10 depicts the outcomes of this deployment.

Figure 10

(a) Input Signal, (b) Random Noise Signal, (c) Output Signal of PID Controller, (d) Filter Output Signal for 50Hz Square Wave Input to VCVB Filter



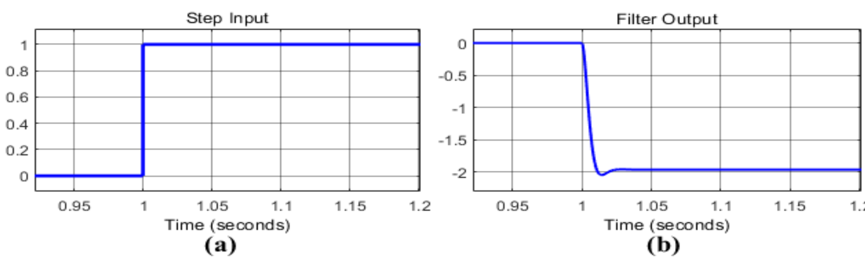
The controller improved the input conditions by using a PID controller in combination with the filter input. This adjustment enabled the filter to provide an enhanced output that is similar to the first filter output. It is shown in Figure 10.

3.2. MFB FILTER

The MFB low pass filter was additionally programmed to have a cutoff frequency of 50Hz. Following the set sequence, the second-order MFB low-pass filter received an initial input of a unit step signal. Figure 11 depicts a graphical depiction of the filter's response to this unit step signal.

Figure 11

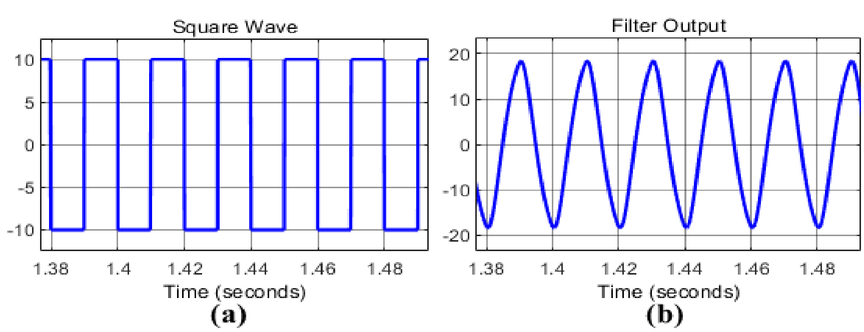
Input and Output Response of MFB Second Order Filter for Unit Step Signal Input. (a) Input Signal, (b) Output Signal



The output of the second-order MFB low pass filter displayed a transient pattern that included negative overshooting when a unit step input was applied. The second order MFB low pass filter was then fed a 50Hz square wave for an actual evaluation. The input and output response waveform for the filter's 50Hz square wave signal is presented in Figure 12.

Figure 12

Input and Output Response of MFB Second Order Low Pass Filter for Square Wave Input. (a) Input Signal, (b) Output Signal



The planned MFB second-order low pass filter was capable of outputting filtered results of a sine wave at 50Hz. The square wave signal is then combined with a random noise signal and transmitted to the filter circuit for real-world testing. Figure 13 depicts the input and output response curve of the filter for a 50Hz square wave signal with random disturbances.

https://doi.org/10.63307/BAJ.4.1.S2

Figure 13

(a) Input 50Hz Square Wave Signal, (b) Random Noise Signal, (c) Square Wave With Noise Added Signal, (d) Filtered Output of MFB Filter

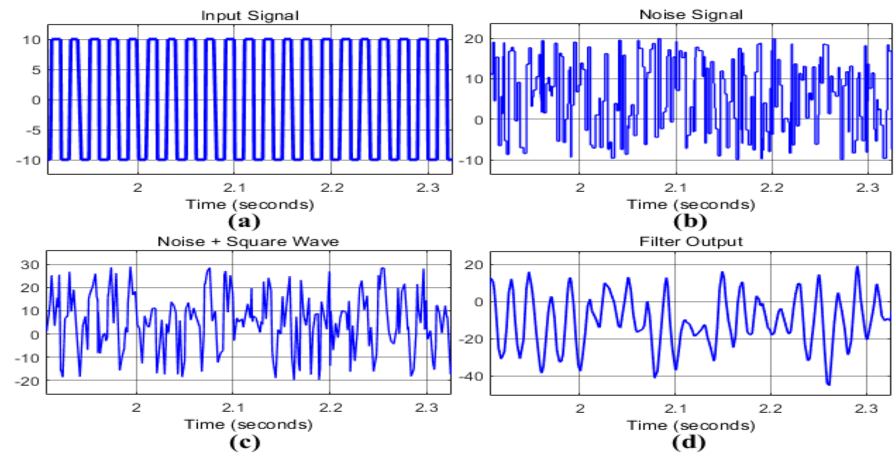


Figure 13 illustrates that when a noise signal is mixed with a square wave and the resulting signal is sent straight to the MFB low pass filter, the filter can't manage to filter the appropriate signal, and the output signal is severely deformed. As a result, to produce the required signal (50Hz sine wave), a PID controller is fitted before the filter to evaluate the output, which has enhanced the output.

Figure 14

(a) Input 50Hz Square Wave Signal, (b) Random Noise Signal, (c) PID Controller's Output Signal, (d) Filter Output Signal of MFB Low Pass Filter

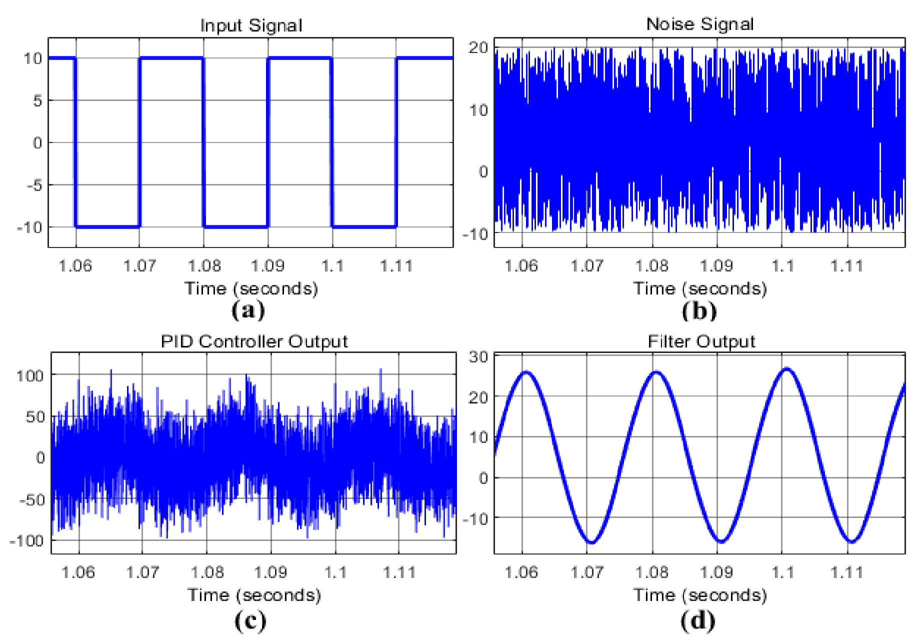


Figure 14 shows that employing a PID controller before the second order MFB low pass filter for noise-added signal assists in resolving the distorted signal issue.

4. Conclusion

Upon conducting a thorough analysis of the results, it can be inferred that the VCVS filter operates in a non-inverting mode, whereas the MFB filter operates in an inverting mode. This implies that utilizing the MFB filter for low gain output would increase the overall cost of inverting the signal again. Further, it was observed that when a noise-added signal was introduced without a PID controller, the VCVS low pass filter produced an output that was 50-80% closer to the desired output as compared to the MFB low pass filter. Conversely, the output of the MFB low pass filter for the same noise-added signal was excessively distorted. However, upon implementing a PID controller, it was observed that the output of the MFB low pass filter was superior and smoother than that of the VCVS low pass filter. Thus, it can be concluded that with the use of a PID controller, the MFB low pass filter can be effectively utilized where cost is not a limiting factor. In contrast, for low-cost operation, the VCVS low pass filter would be a more viable option. Therefore, it is important to carefully consider the cost implications and other relevant factors when selecting a suitable filter for a given application.

References

2nd Order Active Filters, Band-Pass—K7MEM. (n. d.). Retrieved February 26, 2023, from https://k7mem.com/Fil_2nd_Active_BP.html

Badajos, J. C. T., Ochoa, K. M. P., Tejadilla, R. A. P., & Peñas, R. T. L. (2019). Reduction of Audio Noise with Lowpass Chebyshev Type II Filter Simulated using GNU Octave. *2019 IEEE 11th International Conference on Humanoid, Nanotechnology, Information Technology, Communication and Control, Environment, and Management (HNICEM)*, 1–6.

Category: Electronic filter topology. (2022). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Category:Electronic_filter_topology&oldid=1122765651

Efficient Inverse Filters based on Second-Generation Voltage Conveyor (VCII) | *SpringerLink*. (n.d.). Retrieved February 26, 2023, from https://link.springer.com/article/10.1007/s13369-021-05775-5

Harry, J. R., Blinch, J., Barker, L. A., Krzyszkowski, J., & Chowning, L. (2022). Low-Pass Filter Effects on Metrics of Countermovement Vertical Jump Performance. *Journal of Strength and Conditioning Research, 36*(5), 1459–1467.

Nanopower multiple-input DTMOS OTA and its applications to high-order filters for biomedical systems—ScienceDirect. (n.d.). Retrieved February 26, 2023, from https://www.sciencedirect.com/science/article/abs/pii/S1434841120327801

Smith, S. W. (1997). *The scientist and engineer's guide to digital signal processing*. California Technical Pub. San Diego.

Sudhakar, M. V., Charan, M. P., G. N., P., Harika, L., & Yamini, P. (2021). Audio signal noise cancellation with adaptive filter techniques. *Mater. Today Proc*